Intrusion of granitic magma into the continental crust facilitated by magma pulsing and dike-diapir interactions: Numerical simulations

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Abstract We conducted a 2-D thermomechanical modeling study of intrusion of granitic magma into the continental crust to explore the roles of multiple pulsing and dike-diapir interactions in the presence of visco-elasto-plastic rheology. Multiple pulsing is simulated by replenishing source regions with new pulses of magma at a certain temporal frequency. Parameterized *pseudo-diike zones* above magma pulses are included. Simulation results show that both diking and pulsing are crucial factors facilitating the magma ascent and emplacement. Multiple pulses keep the magmatic system from freezing and facilitate the initiation of pseudo-diike zones, which in turn heat the host rock roof, lower its viscosity, and create pathways for later ascending pulses of magma. Without diking, magma cannot penetrate the highly viscous upper crust. Without multiple pulsing, a single magma body solidifies quickly and it cannot ascend over a long distance. Our results shed light on the incremental growth of magma chambers, recycling of continental crust, and evolution of a continental arc such as the Sierra Nevada arc in California.

1. Introduction

Intrusion of granitic magma into continental crust is one of the most important geologic processes contributing to differentiation of the lithosphere and growth of continental crust [e.g., Rudnick and Fountain, 1995; Petford et al., 2000; DeCelles et al., 2009]. Dynamics of magma intrusion have been widely examined through field studies and numerical simulations. Field studies often focus on magma intrusion in a specific setting constrained by field observations [e.g., Paterson and Vernon, 1995; Miller and Paterson, 2001; de Saint-Blanquat et al., 2006; Buenger and Cruden, 2011], whereas numerical studies aim to understand the thermomechanical evolution of the processes [e.g., Bittner and Schmeling, 1995; Annen and Sparks, 2002; Burov et al., 2003; Dufek and Bergantz, 2005; Gerya and Burg, 2007; Keller et al., 2013].

Existing models of magma extraction from its source regions can be broadly divided into two end-member classes: the first assumes that most melt is transported through dikes and the second through diapirc ascent. Diking models assume dike growth in an infinite (visco)-elastic half-space [e.g., Rubin, 1993]. Numerical models of diapirism assume viscous rheology for surrounding rocks [e.g., Bittner and Schmeling, 1995]. Both mechanisms may be active during magma ascent. Miller and Paterson [2001], for example, suggested that growth of plutons was initially accompanied by dikes, which coalesced later and led to diapirc ascent of following magma pulses. To study the transition between these two modes, we employed a numerical code that could handle both end-member scenarios.

Recent field studies on magmatic systems suggest that magmatic systems are usually assembled through multiple increments [e.g., Glazner et al., 2004; Schaltegger et al., 2009; Memeti et al., 2010; Paterson et al., 2011]. However, few numerical studies include multiple intrusions of magma into a thermal model [Barton and Hanson, 1989; Annen and Sparks, 2002; Dufek and Bergantz, 2005] and even fewer thermomechanical models include multiple intrusions [Schubert et al., 2013]. Here we incorporate multiple pulses of magma. Our results show that multiple pulses and diking facilitate ascent and emplacement of magma and may result in a batholith similar to the one of Mesozoic Sierra Nevada in California. Our study more focuses on the dynamics of dike-diapir interaction rather than solving all possible scenarios of magma intrusion (e.g., intrusion of magma of different compositions with various background deformation).
2. Model Description

We employed a finite element marker-in-cell code MILAMIN_VEP [e.g., Crameri and Kaus, 2010; Kaus, 2010; Thielmann and Kaus, 2012], which had benchmarked, versus analytical solutions as well as versus other numerical codes [e.g., Kaus, 2010]. Equations of mass and momentum conservations are solved under a visco-elasto-plastic rheology (see the supporting information). Conservation of energy is given by

$$\rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = \nabla \cdot \left( k \nabla T \right) + H_r$$

(1)

where $\mathbf{v} \cdot (\partial T/\partial x_i)$ is the advection term, $\mathbf{v}$ is the velocity, $C_p$ is the specific heat capacity, $\rho$ is the density, $k$ is the thermal conductivity, and $H_r$ is the volumetric heat production caused by radioactivity. Constant thermal conductivity $k = 3 \text{ W m}^{-1} \text{ K}^{-1}$ and specific heat capacity $C_p = 1000 \text{ J kg}^{-1} \text{ K}^{-1}$ are used for all materials. Table 1 shows abbreviations of variables, and Table 2 shows the thermomechanical parameters of different lithological phases. Since our model does not include temperature-dependent conductivity [Whittington et al., 2009], as well as latent and shearing heating, our model represents a “worst-case” scenario in terms of the cooling time and mobility of magma. We discuss these issues in section 5.
Table 2. Material Properties Used in Numerical Simulations

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho_0 (\text{kg m}^{-3}) )</th>
<th>( T_{\text{solidus}} (\text{K}) )</th>
<th>( T_{\text{liquidus}} (\text{K}) )</th>
<th>Flow law</th>
<th>( E_a (\text{kJ mol}^{-1}) )</th>
<th>( n )</th>
<th>( A_2 (\text{MPa}^{-n} \text{s}^{-1}) )</th>
<th>( H (\text{Wm}^{-3}) )</th>
<th>( C (\text{MPa}) )</th>
<th>( \phi )</th>
<th>( G (\text{GPa}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Felsic crust</td>
<td>2800 (solid) 2500 (molten)</td>
<td>Dinkey Creek biotite granite</td>
<td>Dinkey Creek biotite granite</td>
<td>Wet quartzite</td>
<td>154 2.3</td>
<td>10^{-3.5}</td>
<td>10^{-6}</td>
<td>10</td>
<td>20</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Mafic crust</td>
<td>2900 (solid) 2600 (molten)</td>
<td>Dinkey Creek biotite granite</td>
<td>Dinkey Creek biotite granite</td>
<td>Plagioclase</td>
<td>238 3.2</td>
<td>10^{-3.5}</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>2800 (solid) 2500 (molten)</td>
<td>Wet quartzite</td>
<td>154 2.3</td>
<td>10^{-3.5}</td>
<td>10^{-6}</td>
<td>10</td>
<td>20</td>
<td>25</td>
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<tr>
<td>Lithospheric mantle</td>
<td>3300 (solid)</td>
<td>Dry olivine</td>
<td>532 4</td>
<td>10^{-4.4}</td>
<td>0</td>
<td>10</td>
<td>30</td>
<td>67</td>
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Other properties for all materials: \( \kappa = 10^{-6} \text{m}^2 \text{s}^{-1} \), \( K = 3 \text{W m}^{-1} \text{K}^{-1} \), \( C_p = 1000 \text{J kg}^{-1} \text{K}^{-1} \), \( \alpha = 3 \times 10^{-5} \text{K}^{-1} \), \( \beta = 1 \times 10^{-11} \text{ Pa}^{-1} \).

2.1. Geometry and Boundary Conditions

Our model setup consists of a 200 km x 80 km part of the lithosphere that has a layered crust and mantle (Figure 1). The upper and lower crusts are each 20 km thick, and the lithospheric mantle is 40 km thick. A magma source region is located beneath the crust. The grid has 301 x 301 nodes, which gives a resolution of \( \sim 0.7 \times 0.3 \text{ km} \) per element. The upper and lower thermal boundary conditions are isothermal (surface temperature is fixed to 25°C), and the lateral boundaries are flux free. The mechanical boundaries are free slip for the lower and side boundaries and stress free at the top boundary, which allows topography to develop.

2.2. Partial Melting

Following Gerya and Burg [2007], melt volume fraction (\( M \)) is linearly interpolated between the solidus (\( T_{\text{solidus}} \)) and the liquidus (\( T_{\text{liquidus}} \)) (equation (2)), which are both temperature-pressure dependent. Since we are interested in granitic magma, we use available experimental data of continental arc granite (Dinkey Creek biotite granite in Sierra Nevada, California) [Piwinskii, 1968; Stern et al., 1975] for solidus and the liquidus of felsic crust and magma (see the supporting information for \( P-T \)-solidus and \( P-T \)-liquidus relationship).

\[
M = \begin{cases} 
0, & T \leq T_{\text{solidus}} \\
\frac{T - T_{\text{solidus}}}{T_{\text{liquidus}} - T_{\text{solidus}}}, & T_{\text{solidus}} < T < T_{\text{liquidus}} \\
1, & T \geq T_{\text{liquidus}}
\end{cases}
\]  

(2)

The density of a partially molten rock (\( \rho_m \)) is given by equation (3) [Gerya, 2010]:

\[
\rho_m = \rho_{\text{solid}} + M(\rho_{\text{solid}} - \rho_{\text{molten}})
\]  

(3)

where \( \rho_{\text{molten}} \) is the density of molten rock (Table 2). Density of solid rocks is a function of temperature and pressure [Gerya, 2010]:

\[
\rho_{\text{solid}} = \rho_0 [1 - \alpha(T - T_0)] [1 + \beta(P - P_0)]
\]  

(4)

where \( \rho_0 \) is the reference density at \( T_0 = 298 \text{ K} \) and \( P_0 = 0.1 \text{ MPa} \). \( \alpha \) is the thermal expansion coefficient, and \( \beta \) is the compressibility coefficient. \( \alpha = 3 \times 10^{-5} \text{K}^{-1} \), \( \beta = 1 \times 10^{-11} \text{ Pa}^{-1} \) are used for all materials [Gerya, 2010; Turcotte and Schubert, 2014].

2.3. Viscosity of Magma and Partial Molten Rocks

The magma and partial molten rocks are treated as a Newtonian material of constant viscosity of \( 10^{17} \text{ Pa s} \). This is larger than the real viscosity of felsic magma and partially molten rocks (\( \sim 10^2 - 10^4 \text{ Pa s} \)) [e.g., Dingwell, 2006; Pettford, 2009]. The reason is that similar to most other geodynamic codes, our code employs direct solvers and for numerical reasons (roundoff errors while numerically solving the governing equations) and we have to use both an upper and a lower cutoff viscosities which should not vary more than 6 or 7 orders of magnitude. As a result, we cannot resolve the internal dynamics correctly and will underestimate the flow velocities within the
2.4. Initial Geothermal Gradients

We test three types of initial geothermal gradients. For the first type, we employ a linear geothermal gradient (11.8°C/km) throughout the crust and a Moho temperature of 500°C. We call this a “cold” geothermal gradient, which approximates thermal conditions of a continental crust in the absence of magmatism.

For the second type, a two-stage geothermal gradient is employed. Temperature increases linearly from 25°C at surface to 400°C at 10 km, forming a steep 37.5°C/km gradient. Below that depth, it increases linearly to 900°C at Moho depth, forming a 16.7°C/km gradient. We call this a “hot” geothermal gradient, which approximates thermal conditions in a magmatically active arc crust, constrained by thermobarometric data from paleo-continental arcs [Rothstein and Manning, 2003].

For the third type, a two-stage geothermal gradient is also employed. Temperature increases linearly from 25°C at surface to 400°C at 10 km, forming a steep 37.5°C/km gradient in the shallow crust, below which it increases linearly to 500°C at Moho depth, with a shallow 3.3°C/km gradient. We call this a “warm” geothermal gradient, which can be viewed as an intermediate state between the first two types of geothermal gradients.
2.5. Magma Generation and Multiple Pulses

Conventional model of genesis of continental arc magma suggests that felsic to intermediate arc magma is generated by interaction between basaltic melts from lithospheric mantle and the lower crust in the MASH zone (melting, mingling, mixing, assimilation, storage, and homogenization) [Hildreth and Moorbath, 1988; Annen et al., 2006]. We adapt this model by locating the magma source region right beneath the Moho. New magma pulses are repeatedly generated at the source region with a given time interval to simulate a magmatic flux by simply changing the lithological markers within the source region to magma markers. The first pulse of magma is generated at 0 Myr. Initial temperature of magma is set to 900°C, and its melt volume fraction is 100%. Since we are only interested in how magma ascend and emplace in the crust, the chemical-physical processes of magma petrogenesis are not considered and mass is not conserved in the source region in this study.

2.6. Pseudo-dike Zone Algorithm

The actual physics of diking is complicated, and it involves poorly constrained parameters of fracture mechanisms at the dike tip, stress field in the host rocks, and magma flow [e.g., Rubin, 1993, 1998; Gudmundsson, 2006]. In addition, the real width of dikes (~1–10 m) is below the resolution of our model (~0.7 × 0.3 km per element). Thus, we developed a pseudo-diking algorithm as a first-order approximation of real dikes to simulate their effects on weakening the host rocks by either increasing their average temperature or by advecting magma into the rocks. A pseudo-dike zone is defined as a rectangular thermal anomaly compared to surrounding host rocks. Within a pseudo-dike zone, small dikes are separated by rafts of host rocks (Figure 2). Using 1-D analytical solution of cooling of an error-function-type dike (see the supporting information), we show that the thermal effect of a pseudo-dike zone can be numerically simulated as a single thermal zone with a temperature \( T_{\text{dike zone}} \) given by

\[
T_{\text{dike zone}} = \frac{T_{\text{dike}} N_{\text{dike}} W_{\text{dike}} + T_{\text{host rock}} (N_{\text{dike}} - 1) S}{W_{\text{dike zone}}} 
\]  

(5)

where \( T_{\text{dike zone}} \) is the pseudo-dike zone temperature. We assign temperature of a single dike \( T_{\text{dike}} \) as the average temperature of the molten magma chamber. \( N_{\text{dike}} \) is the number of small dikes in a pseudo-dike zone. \( W_{\text{dike}} \) is the width of a small dike. \( T_{\text{host rock}} \) is the temperature of host rocks. \( S \) is the space between two adjacent small dikes. \( W_{\text{dike zone}} \) is the width of a pseudo-dike zone. To further simplify equation (5), we introduce parameter \( R_M \) to describe the areal fraction of magma in a pseudo-dike zone, and \( T_{\text{dike zone}} \) can be expressed as

\[
T_{\text{dike zone}} = T_{\text{dike}} R_M + T_{\text{host rock}} (1 - R_M) 
\]  

(6)

Two styles of pseudo-dike zone are simulated. Style 1 has a height of 10 km and width of 2 km. Style 2 has a height of 5 km and width of 2 km. A vertical pseudo-dike zone originating from a critical point will be imposed on the solid host rocks, if the following criteria are met:
1. A critical point exists where the maximum of the second invariant of the stress tensor ($\sigma_{II}$) along the interface between solid host rocks and the molten magma chamber is larger than a given critical stress ($\sigma_T$) ($\sigma_T$ is given in Table 3).

2. We compare the area of the magma in chamber with the area of magma in a pseudo-dike zone. If the former one is larger than the latter one, there will be sufficient magma to form a pseudo-dike zone.

To conserve energy, we lower the temperature of the molten magma chamber accordingly after pseudo-dike zones are generated. The temperature drop in magma chamber ($\Delta T < 0$) is calculated according to the following equation:

$$\Delta T = \frac{\text{Area of magma in pseudo-dike zone}}{\text{Area of magma in chamber}} \left( T_{\text{host rock}} - T_{\text{dike}} \right)$$  \hspace{1cm} (7)

In reality, if the crust is homogenous and there are no preexisting weak layers (cleavages, faults, and beds), the orientation of the dikes above a magma chamber should follow the ambient stress field (i.e., dikes should be perpendicular to $\sigma_3$ direction and radiate from the magma chamber in various directions) [Gudmundsson, 2006]. Since our main interest in the study is how diapir and dike interact, we assume that dikes are vertical. The vertical dikes actually might be common if the host rocks have vertical planes of anisotropy (e.g., subvertical cleavages, rotated faults, and beds) just like the host rocks in central Sierra Nevada [e.g., Tobisch et al., 2000; Paterson et al., 2014; Cao et al., 2016].

3. Results

3.1. Intrusion of Magma Facilitated by Diking and Multiple Pulses in a Cold Crust

Figure 3 shows the results of reference simulation in which multiple intrusions of granitic magma ascend in a cold crust facilitated by pseudo-dike zones (simulation CC_3_4_2; see Table 3 for details). At 1.2 Myr, the second magma pulse (the first pulse is added at 0 Myr) rises to the top of
the chamber, forming two projecting parts, and a pseudo-dike zone initiates at the left prong (Figure 3a). At
2.1 Myr, the magma chamber is still constrained within the lower crust and a pseudo-dike zone continues to
form (Figure 3b). The magma chamber migrates upward once the overlying crust is sufficiently heated and
rheologically weakened. At 6.1 Myr, the magma forms a 10 km × 40 km chamber ponding in the middle part
of the crust (Figure 3c). Several pseudo-dike zones intrude into the upper crust. The viscosity of the upper
crust surrounding the magma chamber is lowered by ~3–4 orders of magnitude due to the emplacement
of pseudo-dike zones. At 8.0 Myr, the new magma pulse initially rises rapidly in the form of diapirs within
the host rocks of low viscosity (Figure 3d). Magma and plutonic materials flow downward at the two sides
of the rising magma.

If we instead employ shorter pseudo-diking zones (5 km × 2 km) and an $R_m$ value of 1, simulation results are
similar (CC_3_4_1; Figure 4). We also conducted a simulation in which the pulsing interval is 2 Myr (CC_3_2_2;
Figure 5). In this simulation, it takes longer time (10 Myr) for magma to reach 20 km depth comparing to the
reference simulation (Figure 3).
In summary, multiple pulses of magma gradually move upward by taking advantage of preheated pathways generated by pseudo-dike zones and earlier pulses of magma. Later magmatic pulses are able to rise as a diapiric body and move much faster. The choice of the height of pseudo-dike zone (10 km or 5 km) does not significantly affect the ascent rate of magma, while lower pulsing frequency will cause slower vertical growth of the magma column (see discussion).

### 3.2. Multiple Pulses Without Dike Zones

We conducted a series of simulations without pseudo-dike zones. We also varied the radius of the magma pulses, but kept the same 1 Myr pulsing interval. Figure 6 shows the simulation results of magma pulse with 3 km (CC_3_5_1), 5 km (CC_3_5_2), 7.5 km (CC_3_5_3), and 15 km radius (CC_3_5_4), respectively. When the pulse size is small (3 km and 5 km), magma cools rapidly and crystallizes completely before the next pulse. Even after 21 pulses and 16 pulses, the magma chambers remain completely below the Moho (Figures 6a and 6b).
When the pulse size is larger (7.5 km and 15 km), repeated magma pulsing heats the crust above the source region and lowers its viscosity (Figures 6c and 6d). As a result, later pulses of magma are able to intrude the preheated lower crust through a narrow partial molten channel (see melt volume fraction plots in Figures 6c and 6d). The magma column grows in size as more pulses are injected into the chamber and magma starts to solidify from the host rock roof and walls. Since no pseudo-dike zones are incorporated in these simulations, the ponding magma is not able to penetrate upper crust.

There is a trade-off between the pulsing interval and the characteristic cooling time of a magmatic body of a radius $R$. If a magmatic body crystallizes before a second body intrudes, it will stall below the Moho; if a new pulse arrives before the first one crystallizes, it has the potential to intrude in the lower crust. Since cooling occurs by conduction, cooling time can be estimated by $t_{\text{cool}} = R^2 / \kappa$, where $\kappa$ is the thermal diffusivity (10^{-6} m^2/s) and $R$ is the radius of the magmatic body. If the time between pulses is $t_{\text{pulse}}$, intrusion in the lower crust is possible if $t_{\text{cool}} > t_{\text{pulse}}$. For $t_{\text{pulse}} = 1$ Myr, a magma body with a radius larger than ~5.6 km is needed to intrude in the lower crust, which is consistent with our simulation results (Figures 6c and 6d).

Figure 5. Simulations of 2 Myr pulsing interval (CC_3_2_2). Magma takes longer time to ascend to the base of upper crust comparing to the reference simulation.
3.3. Single Magma Pulse With Pseudo-dike Zones

We conducted a series of simulations only with pseudo-dike zones and without multiple pulses (Figure 7). When the size of the magma source region is small (3 and 5 km; CC_3_3_1 and CC_3_3_2), after generating a few pseudo-dike zones, the magma rapidly crystallizes due to a lack of continuous heat supply by fresh magma (Figures 7a and 7b). As the entire magma plumbing system quenches, upward motion of magma is halted. When the magma source region is larger (7.5 and 10 km; CC_3_3_3 and CC_3_3_4), magma penetrates the bottom part of the lower crust before crystallizing (Figures 7c and 7d). These simulation results suggest that multiple pulses are crucial to maintain long-lived molten magma chamber, the upward growth of the magma column, and the development of upper level magma chambers.

3.4. Influence of Crustal Geothermal Gradient

We also performed simulations within a warm crust and a hot crust. In the warm crust, the first two pulses of magma rise quickly as diapiric bodies (Figure 8). When the magma reaches the upper crust at ~20 km, diapiric ascent slows down and pseudo-dike zones start to form. The magma pulses reach 20 km depth within 1.1 Myr.
In the warm crust, while the magma is still trapped in the lower crust at 1.2 Myr in the cold crust (Figure 3a), the roof of the magma chamber in warm crust also reaches 15 km depth within 6.1 Myr, comparing to 8 Myr in the cold crust for the magma to reach the same depth (Figure 3d).

Significant ductile downward flow of host rocks occurs along the vertical sides of the magma chamber in the warm crust. Upper crustal host rocks are dragged ~20 km down to the magma source region through narrow zones adjacent to the magma chamber (Figure 8d) [Cao et al., 2016]. Beds of host rocks are rotated and curved, and their thicknesses are reduced. The downward ductile flow of host rocks in the warm crust is induced by shear stress imposed by the downward return flow within the magma chamber. The downward ductile flow of host rocks is favored by a lower viscosity contrast and thus an increased coupling between the magma chamber and host rocks, which is ultimately related to the higher geothermal gradient.

In the hot crust simulation (CC_3_4_3) (Figure 9), lower crust shows extensive melting and vigorous convection due to Rayleigh-Taylor instability [Sharp, 1984], and the upper crust forms drips falling into the molten lower crust (Figure 9a). As soon as upper crustal materials reach the Moho, they extend laterally before melting and becoming assimilated with the surroundings (Figures 9b and 9c). Only a few pseudo-dike zones are generated in the hot crust simulation.

3.5. Influence of Rheology of Lower Crust

We conducted a simulation with a strong mafic lower crust, which employs a plagioclase rheology instead of a wet quartzite rheology used in other simulations (CC_3_6_1; Figure 10) and a short pulsing interval (0.3 Myr). The overall dynamics is similar to the simulation with a weaker lower crust (e.g., Figure 3). The differences are as follows: The magmatic system with a strong lower crust has a more rectangular shape and does not show significant lateral spread. The pseudo-dike zones are the dominant mechanism to heat...
host rocks and guide magma upward in the cold and strong lower crust, where diapirism is unable to operate outside of the magma column. Host rocks are sharply truncated by the magma chamber showing no ductile bending and downward flow. In contrast, in the simulations with a warm felsic lower crust (Figure 8), diapirism is the main mechanism of magma ascent at lower crustal and the magma chamber has a typical “mushroom” shape.

4. Dimensional Analyses

4.1. Lateral Construction of Magma Chamber

We estimate the rate of lateral growth using a simplified model (Figure 11), in which the effective viscosity of the host rocks is \( \eta_{\text{host}} \), the density contrast between magma chamber and host rocks is \( \Delta \rho \), the ascent velocity of the magma chamber is \( v_z \), and the lateral growth velocity of magma chamber is \( v_x \). We approximate that the magma chamber as a cylinder, whose radius is \( r_1 \), height is \( h_1 \), and its volume is \( V_{\text{chamber}} = \pi r_1^2 h_1 \).
\(v_z/h_1\) gives the approximate strain rate of magma chamber along \(z\) direction. The ascent velocity \((v_z)\) can be estimated in the following equation if the buoyancy force is balanced with viscous stress applied on the surface of the cylinder [Davies, 1999, modified from equation (6.8.3)]:

\[
\Delta \rho_1 g (\pi r_1^2 h_1) = \frac{\eta_{\text{host}} v_z}{h_1} \left(2\pi r_1 h_1 + 2\pi r_1^2\right)
\]

(8)

Mass conservation relates \(v_x\) and \(v_z\) when the magma chamber reaches a high-viscosity roof and the top of chamber stops moving upward:

\[v_x 2\pi r_1 h_1 = v_z \pi r_1^2\]

(9)

Combining equations (8) and (9) gives the lateral growth velocity:

\[v_x = \frac{1}{4} \frac{\Delta \rho_1 g r_1^2 h_1}{\eta_{\text{host}}} \left(\frac{1}{h_1 + r_1}\right)\]

(10)

Equation (10) suggests that the lateral velocity inversely depends on the viscosity of host rocks. If \(\Delta \rho_1 = 300 \text{ kg/m}^3\), \(r_1 = h_1 = 10 \text{ km}\), and effective viscosity of host rocks surrounding the magma chamber ranges from \(10^{20}\) to \(10^{21} \text{ Pa s}\) as shown in simulated viscosity diagrams, then \(v_x = 1.2\) to \(12 \text{ km/Myr}\). The lateral growth rate in most of our simulations fall within this ranges. Gerya and Burg [2007] showed that lateral spreading of magma occurs if host rocks have a lower viscosity while the magma chamber forms a

Figure 9. Simulation of a hot crust (CC_3_4_3). Extensive melting occurs in the lower crust accompanied by vigorous convection. Upper crust materials drip into the molten lower crust.
constrained channel if host rocks have a higher viscosity. Their results are consistent with our analysis. We can estimate the horizontal length \( L_{\text{chamber}} \) of a magma chamber by

\[
L_{\text{chamber}} = L_0 + v_x \cdot t_{\text{growth}}
\]

(11)

where \( t_{\text{growth}} \) is the growth time of the magmatic system. For a chamber constructed by multiple pulses, the growth time depends on pulsing numbers and frequency and the growth time can be several million years. \( L_0 \) is the initial horizontal length of the chamber. \( L_0 = 0 \) if the magma chamber starts from intrusion into host rocks from elsewhere. To justify the dimensional analysis, we can use equation (11) to back-calculate the longer-term host rock effective viscosity if we know the lateral size and construction time of an intrusive complex. For example, Tuolumne Intrusive Complex in California was constructed in about 10 Myr and its longer dimension is about 55 km [e.g., Memeti et al., 2010]. If we assume that \( \Delta \rho_1 = 300 \text{ kg/m}^3 \), \( r_1 = h_1 = 10 \text{ km}, \) and \( L_0 = 0 \). We calculate \( v_x = 5.5 \text{ km/Myr} \) based on equation (11) and again use equation (10) to back-calculate the host rock effective viscosity \( \eta_{\text{host}} \approx 2.2 \times 10^{20} \text{ Pa s} \), which is close to the effective viscosity of host rocks \( (10^{20} \text{ to } 10^{21} \text{ Pa s}) \) surrounding the magma chamber predicted in the simulations.

**Figure 10.** Simulation of a strong lower crust of plagioclase rheology (CC_3_6_1) and a mafic composition. Magma chamber has a more rectangular shape, and host rocks are sharply truncated by the magma chamber with no ductile bending.
We apply the same rationale within a magma chamber when a new magma pulse is just injected (Figure 11). We assign the effective viscosity of the magma chamber as $\eta_{\text{chamber}}$. Density contrast between a new magma pulse and the existing magma chamber is $\Delta \rho_2$. The ascent velocity of the new magma pulse is $u_x$, and the lateral spreading velocity of new magma pulse is $u_x$. The new magma pulse is treated as a cylinder whose radius is $r_2$, height is $h_2$, and volume is $V_{\text{pulse}}$. Similar to equation (8), the horizontal velocity of the new magma pulse within the existing magma chamber is

$$u_x = \frac{1}{4} \frac{\Delta \rho_2 g r_2^2 h_2}{\eta_{\text{chamber}}} \left( \frac{1}{h_2 + r_2} \right)$$

(12)

Since $\eta_{\text{chamber}}$ is several orders of magnitude less than $\eta_{\text{host}}$, the lateral spreading velocity should be $u_x \gg v_x$. The result is that the flow of a new magma pulse has to form return flows when it reach the walls of the magma chamber and at the same time pushing existing materials in the magma chamber downward as part of the intrachamber return flow. Many of our simulations show such return flows within the magma chamber (e.g., Figures 3, 4, 5, 8, and 10).

4.2. Factors Controlling the Ascent of Magma

In a viscous environment where diapiric ascent is allowed, the ascent velocity of magma ($v_{\text{diapir}}$) can be estimated using Stokes velocity [Turcotte and Schubert, 2014]:

$$v_{\text{diapir}} = \frac{2}{9} \frac{\Delta \rho r_1^2}{\eta_{\text{host}}}$$

(13)

For $\Delta \rho_1 = 300 \text{ kg/m}^3$, $r_1 = 10 \text{ km}$, $\eta_{\text{host}} = 10^{20} \text{ Pa s}$, $v_{\text{diapir}}$ is about $21 \text{ km/Myr}$. This velocity matches the diapiric velocity in the warm lower crust as shown in Figure 8a, in which magma ascends from 40 to 20 km depth within 1.1 Myr. When the magma reaches a high-viscosity host rock roof, pseudo-dikes will channel magma upward. The pipe-flow velocity ($v_{\text{pipe}}$) of magma in a pseudo-dike zone can be estimated by an equation modified from equation 6.34 in Turcotte and Schubert [2014]:

$$v_{\text{pipe}} = \frac{1}{4} \frac{\Delta \rho g w^2}{\eta_{\text{magma}}}$$

(14)

For $\Delta \rho_1 = 300 \text{ kg/m}^3$, $w = 1 \text{ km}$ (half-width of a pseudo-dike zone), $\eta_{\text{magma}} = 10^{17} \text{ Pa s}$, $v_{\text{pipe}}$ is about 0.24 m/yr, and thus significantly faster than the $v_{\text{diapir}}$. With such velocity, it takes less than 10 kyr for the magma to reach the top of a 10 km high dike zone. In other words, the response time of pipe flow is significantly short. Therefore, when the magma reaches the high-viscosity roof, its ascent velocity is controlled by the ability to have pseudo-dike zones, which depends on the stress conditions along the contact. Since we use visco-elastic-plastic rheology for the host rocks, it is not straightforward to derive a closed form analytical solution for the stress in the host rock roof, and thus the conditions for pseudo-dike generation. We therefore quantified the ascent velocity from our simulation results. Figure 12 shows the depth of magma chamber roof versus pulse numbers of four different simulations. Although dike heights, pulsing intervals, and flow laws vary in these simulations, the general trend is consistent. The $v_x$ can be approximated as

$$v_x = \frac{\xi}{\Delta t_{\text{pulse}}}$$

(15)

where $\Delta t_{\text{pulse}}$ is the pulsing interval and $\xi$ is a constant length ($\xi = 2.7 \text{ km}$) based on Figure 12. While the inverse relationship between $v_x$ and $\Delta t_{\text{pulse}}$ is expected, the constant $\xi$ is likely caused by the fact that we...
used the same magma radius (10 km) in all simulations. Stress should, to first order, be positively related to the pulse size. Whereas the ascent velocity is more or less constant, the magma chamber will laterally expand as the top of the magma system is usually partially melted and the lateral size of chamber can grow for a longer time according to equation (11). The result is that the magma chamber grows into a “T” shape in many simulations (e.g., Figures 3, 4, 5, and 8).

5. Model Limitations

We did not include the latent heat and temperature-dependent thermal conductivity in the model; time span of a molten magma chamber were underestimated. Gelman et al. [2013] suggested that if latent heat, temperature-dependent thermal conductivity, and nonlinear relation between temperature and melt fraction were all included, the magma chamber would have 100–200% increase of the volume of mobile magma (melt volume fraction > 60%) and twice longer cooling time. If magma can stay molten for a longer time, there will be more chances for magma to form as a diapiric body, generate dikes, and ascend in a faster way.

Our simplified diking algorithm did not include pore fluid pressure, which could be generated by magmatic volatiles or pore fluids in host rocks [Lister, 1990; Rubin, 1993]. The pore fluid pressure could help to reduce the tensile strength in the host rocks and facilitate fracture propagation [Rubin, 1998]. Diking process will be more active, and it could fasten the upward migration of magma if pore fluid pressure is included. We also assume that all the pseudo-dike zones are vertical when they are generated. In reality, dike orientation can vary depending on the stress field in host rock. Our model thus may overestimate the ascent velocity of magma when it is channeled upward by dikes, which may counteracts the effect of pore fluid pressure.

We have ignored the petrogenesis of magma as well as melt segregation and extraction required to gather melts before they start to ascend [e.g., McKenzie, 1985; Pettford et al., 2000]. Fine-scaled simulations with melt-solid rock system [e.g., Keller et al., 2013] and phase transitions are needed to investigate how these processes affect the magma generation. Our model also employed a fixed magma source region in a neutral tectonic environment. In reality, magma generation rate, the size of magma pulse, and the exact location of magma source regions could vary temporally and spatially based on availability of fertile materials for magma generation, thermomechanical conditions of lithospheric mantle, formation or foundering of an arc root, and crustal thickness [e.g., Ducea et al., 2015; Profeta et al., 2015]. Multiple magma source regions are likely to exist and potentially provide multiple pathways for magma to ascend and move horizontally promoting mixing/mingling processes. A future study should include multiple source regions and tuning to magma size and pulsing frequency to magmatic fluxes observed in the continental arcs [Paterson and Ducea, 2015] and include background deformation.

6. Geological Relevance

Our simulation results shed light on the ascent and emplacement of felsic magma in continental arcs. Heating crust through multiple dikes or sills was tested in previous models [e.g., Dufek and Bergantz, 2005; Annen, 2011]. Hardee [1982], Paterson and Miller [1998], and Miller and Paterson [2001] proposed incremental magma ascent and growth models involving linked diking and diapir mechanisms. They argue that early magmatic dikes...

Figure 12. A plot showing that the depth of magma chamber roof decreases with every additional magma pulse. The average ascent velocity (2.7 km/pulse) is more of less constant for different simulations when depth is plotted with pulse number. Colors of curves refer to different simulations: red = CC_3_4_2, blue = CC_3_6_1, black = CC_3_4_1, purple = CC_3_2_2. The insert box shows the height of pseudo-dike zone, pulsing interval, and lower crust rheology used in simulations. wet qtz. = wet quartzite, Plag = Plagioclase An75 (see Table 2).
intrude the host rocks above the magma chamber, after which the dikes coalesce and host rocks raft form between dikes. These dikes serve as a preheated pathway for later batch of magma to rise in a diapirc form [Miller and Paterson, 2001]. Our simulation results portrait a very similar dynamics and support their models. The diking-facilitated magma ascent may also play a role in magma ascent in the Sierra Nevada arc, where host rocks show steeply dipping bedding and rotated thrust faults [Schweickert and Lahren, 2006; Tobisch et al., 2000; Cao et al., 2015]. We envision that these subvertical anisotropic zones in host rocks may serve as a preferential place for dikes to initiate, which is consistent with the numerical simulations by Das et al. [2014], which shows that the lithological contacts serve as zones of anisotropy, and they are more prone to localize strain, become fractured and faulted, and facilitate the magmatic flow. Large-scale transpressional faults in upper brittle crust may also from multiple pathways facilitating magma intrusion [e.g., Tikoff and Teyssier, 1992].

Our simulations support the existence of large, partially molten, long-lived magma chambers in active continental arcs. Several simulations (e.g., Figures 3, 4, 5, and 8) suggest that partially molten magma chambers of 10–20 km in radius ponding at ~20 km depth. Such results may reflect the observations of anomalies of seismic velocities, electrical resistivity, and gravitation in the modern Andes. The size of the anomalies range from 10 to 100 km in radius and 5 to 10 km in thickness, and they pond at ~5–25 km depth. They are interpreted to be active mush-rich magma reservoirs underneath surface volcanoes [e.g., Potro et al., 2013; Singer et al., 2014; Ward et al., 2014].

The resided magma chambers in the crust also provide a temporal period needed for magma to evolve geochemically and potentially erupt [Brown and Fletcher, 1999; Davidson et al., 2007]. Growing lines of evidence from field studies, geochronology, and single mineral chemistry suggest that magma mixing and host rock assimilation play important roles in the structural and compositional evolution of magma and plutons [e.g., Miller et al., 2007; Davidson et al., 2007; Paterson, 2009]. Our simulations show magma mixing, recycling, and host-rock incorporation during magma chamber construction, that may to some extent mimic the real dynamics in the magma chamber.

Our results also provide insights into the downward flow of host rock. The downward transfer of host rocks has been proposed by Saleeby [1990] and Paterson and Farris, 2008 to solve the “space problem” during pluton emplacement. In the Sierra Nevada, lines of evidence for downward transfer of host rocks include (1) plutons intruding older volcanic rocks or sedimentary rocks transported downward to emplacement level [Saleeby et al., 1990; Cao et al., 2016], (2) folding of bedding adjacent to plutons [Paterson and Farris, 2008], and (3) subvertical stretching lineation in host rocks [Paterson et al., 2014; Cao et al., 2016]. Our simulations with a weak lower crust (Figures 3 and 4), especially the one with weak warm crust (Figure 8), show that accompanying the diapirc ascent of magma, adjacent host rocks are bended, thinned, transported downward ductily. In the simulations, host rocks at initial 20 km depth are transported to ~40 km depth within ~1–10 Myr, which is consistent with field observations [Cao et al., 2016]. Downward transported host rocks would probably fertilize the magma source regions. Oxygen isotopic results suggested that crustal sources have contributed up to 50% to the Sierran magma [Ducea and Barton, 2007]. The downward transfer of host rocks could be a contributor to this process.

Finally, our simulations with cold, warm, and hot crusts can be viewed as three different stages of a continental arc (Figure 13). During Stage 1 (cold crust), when the arc just starts, crustal geothermal gradient is low. The crust is underplated by basaltic magma and gets partially melted. MASH zone [Hildreth and Moorbath, 1988] is formed as the source regions of granitic magma. As the crust is cold, diapirc ascent of magma is difficult. The dominant style of magmatic intrusion is diking, which shoots melt from the MASH zone into the base of the crust. These dikes warm the crust and reduce its viscosity. During Stage 2 (warm crust), the arc’s geothermal gradient is elevated and the crust become less viscous, allowing diapirc ascent of magma. Dikes are formed at the top of magma pulses, and they in turn form preheated zones and facilitate the diapirc ascent of later pulse of magma. Host rocks adjacent to the ascending magma are transported downward. In the Stage 3 (hot crust), with more magma intruded and continuous basalt magma underplating, the crust becomes hotter and partial melting becomes extensive in lower crust. Such partial melting drives the convection which might be pervasive up to a depth of ~15–20 km. Magma rise as diapirc bodies and host rocks drip downward, and some of them could get melted and recycled in the MASH zone. Magma mush zones pond at 10–20 km depth. Similar crustal convection was proposed by Babeyko et al. [2002] to explain large-scale magmatism in the Altiplano-Puna region of the central Andes and Schenker et al. [2012] to explain migmatitic core.
complex (Naxos dome in Aegean area). We propose that such crustal convection is also likely to occur in the Late Cretaceous Sierra Nevada arc when voluminous magma intruded the crust during the magmatic flare-up events [Ducea, 2001; Paterson and Ducea, 2015].

7. Conclusions

Our simulations show that diking and multiple pulses are two crucial factors that facilitated the ascent and emplacement of granitic magma into upper continental crust from source region located at lower crust-upper mantle. Multiple pulsing of magma continuously provides buoyant, high-temperature, and molten
materials, which is able to keep the magmatic system from freezing. The pseudo-dike zones heat the host rocks and lower the host rock viscosity above the magma chamber and thus localize and channel the later pulses of magma migrating upward. The simulation results support the incremental growth of a large magma chamber at middle-upper crust, magma mixing and host rock assimilation, recycling continental crust by downward transfer of materials to magma source regions, and crustal convection caused by lower crust melting. We also propose a model of three-stage-evaluation of a continental arc as it becomes more and more thermally matured due to magmatism.

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